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# NPV risk simulation of an open pit gold mine project under the O'Hara cost model by using GAs

Franco-Sepulveda Giovanni *, Campuzano Carlos, Pineda Cindy

*Department of Materials and Minerals, National University of Colombia, Medellín 050036, Colombia*

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ABSTRACT

This paper analyzes an open pit gold mine project based on the O'Hara cost model. Hypothetical data is proposed based on different authors that have studied open pit gold projects, and variations are proposed according to the probability distributions associated to key variables affecting the NPV, like production level, ore grade, price of ore, and others, so as to see what if, in a gold open pit mine project of 3000 metric tons per day of ore. Two case scenarios were analyzed to simulate the NPV, one where there is low certainty data available, and the other where the information available is of high certainty. Results based on genetic algorithm metaheuristic simulations, which combine basically Montecarlo simulations provided by the Palisade Risk software, the O'Hara cost model, net smelter return and financial analysis tools offered by Excel are reported, in order to determine to which variables of the project is more sensitive the NPV.



## 1. Introduction

The risk of a mining project is always one thing to consider for investors before deciding to put their money on one [1]. Costing techniques for the mining industry are key tools to perform ex ante NPV analysis on mining projects, whether they are surface or underground, and they are helpful in estimating the cost of different items, like machinery, processing plant, wages for personnel, transport, among other items, in which investors have to think about when saying yes to investing in the project. It is of great help to know, in first place, to which parameters or variables is more sensitive the mining project as a whole, and this point is crucial to be assertive in the final conclusion of the NPV risk analysis [2–5].

Simulating stochastic techniques and tools are also required, like Montecarlo simulation, used in this work, which have been used for several studies in the mining industry and other kind of investment projects in the past [6–10]. A large probability distribution portfolio is also required, to analyze data and adjust to best fit density distributions both, parameter and objective functions. For that matter, Palisade Risk 5.5 Industrial was used.

Genetic algorithms (GAs) concept is used to implement a combination of the named methodologies, which turns into heuristics, for which GAs is a metaheuristic used for optimizing the NPV of the gold mining project of this case of study, which is one that extracts 3000 tons per day of ore [11–16]. Terms like parameter and input, objective function and output, are interchangeable within this paper, and they relate with the optimizing problem of the NPV which is aimed to be solved.

Being the price of gold, the most important parameter to affect the NPV of the project, further deep studies have to be made, in order to know the behavior of this volatile parameter. Though volatile, real gold prices have cycles, as any other commodity does [17–20]. For instance, the dow/gold ratio has been found to be cyclic, and even more, that it has an average cycle time of 37 years [19,21].

The second important variable to affect the NPV is the mean ore grade, for which a geological exploration has to be carried out with good care in the mine object of risk analysis. Techniques of distribution of ore grade estimation within the subsoil, like Krigging and other new techniques like neural networks and fuzzy logic, among others, are to be studied to understand how to process the logged geological data and report it. So the quality persons (QPs) can check that everything is being done according to the accepted international standards like NI 43/101, JORC and SAMREC, also concepts of cutoff grade are mentioned, in order to understand the complexity of analyzing the risk level of a mining project in terms of its NPV [1,4,22–27].

Simulation for mining projects has been performed. Dimitrakopoulos and Abdel Sabour compares the real options valuations (ROV) with the static calculation of the NPV, and

* Corresponding author.
*E-mail address:* gfranco@unal.edu.co (G. Franco-Sepulveda).

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established that the ROV based design is worth 11%–18% more than the fix NPV based design, for it accounts the uncertainty of the variables that impact the NPV, and treats it as a stochastic variable, instead of a static one, also, the ROV technique represents a tool to tell as the project goes on, what is the probability of success of making decisions like expanding production capacity, reducing it, or even abandoning the project [28].

Keswani and Shackleton show that the ROV technique is capable of augmenting the NPV of an investment project, because of the flexibility it permits in handling decisions on the fly [29].

Huang remarks the importance of managing uncertainty of the parameters that affect the NPV by using stochastic input data and taking it as a fuzzy variable, instead of just the base of expert's opinions, for it becomes a tool for experts themselves to see beyond their own personal assessments, the reality of the uncertainty that is transmitted to the NPV itself by the volatility of markets, production costs, and other factors involved in an investment project, so as to have a better base to assign capital budgeting for them [30].

Leite and Dimitrakopoulos proposed a method of scheduling a mining project under uncertainty of copper ore/metal supply that improves the conventional mine scheduling used in that industry, for it increases significantly the NPV. The ore supply is not constant in any mining project, and the fact that a processing plant is a capital with fixed and variable costs to be minimized with respect to every unit produced over the life of mine. This variable becomes an important issue, for both technical and geological uncertainties may come up and damage a mining project economically, and planning for avoiding problems about this is very important [31].

Van Groenendaal suggests that modeling the NPV as a stochastic variable from known deterministic models, and thus, by using a regression modeling techniques, improves the modeling of the uncertain behavior of the NPV, thus, determining its variability [32]. This variability, in the end, is the ultimate influential factor in deciding to go or not in an investment project.

It is important to say that in this paper, the NPV is taken as a stochastic dependable variable, for the uncertainty is taken into account for the input variables to affect it, and the only thing that is not covered in this work, is the fact of making decisions on the fly for the project, as the ROV technique allows to, so it is suggested for future research.

This paper aims to find which variables impact the most an open pit gold mining project gold of 3000 tons per day of ore, which has 3 years to build up its infrastructural base and ten years more to extract the mineral, process it and sell it, and to which extent they do, so as to have an experience that allows us to decide how to maximize the NPV of a project on the run like the described one based on an ex ante study, and under uncertainty [33]. It is done so, by using the deterministic O'Hara costs model taking its input variables as stochastic ones, taking the income also as a stochastic variable dependent on gold prices and other factors, which are taken into account in other deterministic model that calculates the net smelting return, to finally by means of a Montecarlo simulation assess the variability of the NPV under low and high certainty scenarios of ex ante information available [34]. This whole arrangement of mathematical and computational techniques working altogether fulfills the criteria of a genetic algorithm (GA).

## 2. Theoretical framework

### 2.1. O'Hara cost model

#### 2.1.1. Cost modeling techniques

A cost model for the mining industry can be built by using different methodologies. There are two basic methodologies to get used to: symmetric and asymmetric cost models [2,6]. The first methodology is a deterministic one, and is based on statistics of many mining projects that have had taken place, and adjusts a curve to pairs of data that relates cost estimators versus parameters like production level for most of the items (personnel required, area for the pit and processing plant, unit cost of production, capital costs, services, etc.), and the least squares criterion is used for adjusting, while the second methodology is a probabilistic one and is based on econometric techniques and even artificial intelligence heuristics like neural networks (NN), fuzzy logic, and meta-heuristics like genetic algorithms (GA) [12,35].

The O'Hara cost model was created under the first named methodology, and therefore it is a deterministic one. The model allows mining engineers and other professionals interested in estimating costs for a mining project to introduce some parameters the model asks for, and then estimate them, and it is also possible to estimate the amount of machines, personnel and space buildings and all infrastructure required for developing the mining project with all the technical requirements.

In most of cases, cost estimators for different items are adjusted by least squares method based on Eq. (1) [6]:

$$C(T) = \alpha T^{\beta} \quad (1)$$

where $C(T)$ is the cost; $T$ the extraction rate in tons or ore per day; and $\alpha$ and $\beta$ the parameters of adjustment calculated by least squares numerical method from statistical data, and depend on the item of the whole cost model that is desired to be estimated.

Models like this are very important for investors, for they can begin to get used to the numbers that the mining industry handles, and see if they own the money to invest in one and/or how can they afford such amounts. But the model itself, because of being deterministic, doesn't tell them how risky the inversion is, and a probabilistic way of thinking needs to be introduced and implemented. Mining projects are complex businesses and demand constant risk assessment. This is because several kinds of uncertainties influence the value of a mine project [1,10].

### 2.2. Up to date O'Hara cost model

#### 2.2.1. Estimating costs

The original O'Hara cost model is made to calculate costs in dollars of 1989 [2,34]. In this paper, because a whole financial model is intended to be created, so as to report costs in dollars of year 2015, a correction factor is introduced in order to achieve this. So, the proposed model for calculating cost estimations is as follows in Eq. (2).

$$C(T) = f * \alpha T^{\beta} \quad (2)$$

where $T$ is the number of daily mined tons of both, waste and ore; and $f$ the named factor, which is calculated based on successive yearly inflation rates in the USA by using Eq. (3).

$$f = \prod_{k=1}^{N} (1 + i_k) \quad (3)$$

where $i_k$ is the inflation rate in year $k$; and $N$ is given by Eq. (4)

$$N = y_k - 1989 \quad (4)$$

where $y_t = 2017$ is the year to which the costs are going to be brought as present values.

The inflation rate series to find the factor $f$ is taken from year 1989 to 2017, and it has been found that 2.029775 which means that in average, for the whole American economy, the costs in nominal money have almost doubled compared to what they were in 1989 [38].

This proposed up to date O'Hara cost model is the base for the whole further NPV analysis presented in this paper.

#### 2.2.2. Estimations

The amount of people, trucks, shovels, people to work in the shovels, trucks, plant and offices, can be estimated in proportion to the daily tons of mined material $T$, as follows according to Eq. (5):

$$N(T) = aT^b \tag{5}$$

where $N(T)$ is any of the named variables; and constants $a$ and $b$ depend on which item of the cost structure is to be estimated.

### 2.3. Net smelter return

The net smelter return (NSR), is an economic concept applied to metallurgical processes that involve metallic enrichment [34]. The NSR will be the cause of income to the mining project, as it can be implied, and it is the key of the success of the project itself. By having the NSR found, the metallurgical incomes and outcomes are all taken into account, and it could be seen as the profits gotten from the metallurgical process, having still to subtract from it the mining costs. The NSR can be calculated by using Eq. (6).

$$NSR_k = M_{e,k} * P_{e,k} - T_{ch,k} - X_k - Y_k - T_{rs,k} - R_{c,k} \tag{6}$$

where $M_{e,k}$ is the metallic content of the processed rock in year $k$; $P_{e,k}$ the effective price of the ore concentrate per tone in year $k$; $T_{ch,k}$ the treatment (royalties) charge in year $k$; $X_k$ the penalty for contaminants in the ore concentrate in year $k$; $Y_k$ the value of recoverable metals in the ore concentrate in year $k$; $T_{rs,k}$ the transportation cost in year $k$; and $R_{c,k}$ the realization cost in year $k$ (covers payments for sellers, ads, etc.).

The effective price of the metal content in rock is dependent on the recovery percentage of the metallurgical process, and it is given by Eq. (7).

$$P_{e,k} = P_k * R_{r,k} - PC_k/M_{e,k} \tag{7}$$

where $P_k$ is the actual market price of the ore concentrate in USD/ton in year $k$; $R_{r,k}$ the recovery rate of the metallurgical process in year $k$; $PC_k$ the processing cost in year $k$; and $M_{e,k}$ the accredited metal content in year $k$.

### 2.4. Equilibrium point

The equilibrium point is the key to motivate an NPV analysis on a mining project. It is defined as the point in which the cost curve and the sales curve cross each other. The cost curve is represented by Eq. (8).

$$C(x) = FC + VC * x_O \tag{8}$$

where C($x$) is the total cost in USD/day; $FC$ the fixed cost in USD/day; $VC$ the variable cost in USD/tonne/day; and $x_o$ the number of produced units per day. The sales curve is represented by Eq. (9):

$$S(x) = Px_D \tag{9}$$

where S($x$) is the worth of the sales in USD per day; $x_D$ the number of sold units per day; and $P$ the price in USD/ton/day.

When the sales equalize the total cost, then it is said that the equilibrium point has been reached, it means, the sales value is just enough to cover up all of the costs. The number of units necessary to sell, in order to reach that equilibrium point can be found by using Eq. (10):

$$x_{eq} = \frac{FC}{P - VC} \tag{10}$$

In order to be profitable, a mining project has to prove it capable to cover a demand of $x_D$ tonnes of ore concentrate per day superior to $x_{eq}$ tons of ore concentrate per day.

This equilibrium point is very much dependent on the level of production the mine undergoes to, and of course, the need of a deep market study is evident, to have clear from this stage of the analysis if the project could be profitable financially.

In Fig. 1, it can be seen the curves calculated by using the O'Hara cost model in Excel, the demand level in a green vertical line, on its left, the equilibrium point, formed by the intersection of the costs and sells curves. In this particular case, the project is able to cover the costs with the sells.

### 2.5. NPV analysis

In order to analyze a gold mine project from the money point of view, it is required a mathematical model, after having that, the output variables to be analyzed have to be established, then input variables will be considered to be investigated in both, the mine and gold market [36].

The output variables for analyzing the cash flow of an open pit gold mine are net present value (NPV), internal return rate (IRR) and weighted average capital cost (WACC).

By means of different equations yet to be seen, there are conditions to affirm that (see Eqs. (11)–(13)):

$$NPV = f\left(\beta, \frac{D_{bt}}{E}, \frac{D_v}{P_{sh}}, r_{gm}, g, P, W_{pfd}, R_r, f_{rr}, SR, D_0, t, T_0, T_w, k_d\right) \tag{11}$$

$$IRR = f\left(\beta, \frac{D_{bt}}{E}, \frac{D_v}{P_{sh}}, r_{gm}, g, P, W_{pfd}, R_r, f_{rr}, SR, D_0, t, T_0, T_w, k_d\right) \tag{12}$$

By means of different equations yet to be seen, there are conditions to affirm that (see Eqs. (11)–(13)):

$$WACC = f\left(\beta, \frac{D_{bt}}{E}, \frac{D_v}{P_{sh}}, r_{gm}, W_{pfd}, R_r, f_{rr}, t, k_d\right) \tag{13}$$

where $\beta$ is the beta of the gold mining market; $D_{bt}/E$ the debt/equity ratio; $D_v/P_{sh}$ the dividend/share price ratio; $g$ the grade of ore; $P$ the market price of gold; $W_{pfd}$ the proportion of the financing taken provided by preferred stock; $R_r$ the recovery rate; $r_{gm}$ the mining industry market risk rate; $f_{rr}$ the risk free rate; $SR$ the stripping ratio; $D_0$ the stripping soil overburden; $t$ the tax rate; $T_o$ the tons of ore mined per day; $T_w$ the tons of waste mined per day; and $k_d$ the debt cost. These output variables are all related to Eq. (14).

$$NPV = -I_0 + \sum_{k=1}^{N} \frac{F_k}{(1 + IRR)^k} \tag{14}$$

where $I_0$ is the initial investment (year zero), which covers everything from the building of the mine phase in the O'Hara cost model; and $F_k$ the cash flow of year $k$.

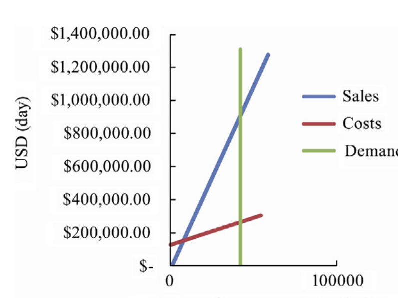


**Fig. 1.** Equilibrium point of the mining project compared with the demand of ore concentrate.

The former is proportional to the production level of the mine, and can be found by Eq. (15):

$$I_0 = MACC + MEC + PSC + CBC + GPCC \tag{15}$$

where $I_0$ is the initial investment of the mining project (year zero); $MACC$ the mine associated capital cost (includes clearing, soil stripping and waste stripping costs); $MEC$ the mining equipment cost; $CBC$ the concentrator building cost; $PSC$ the pit services cost; and $GPCC$ the general plant capital cost.

The cash flows per year are dependent on several factors, both operational and market ones. The cash flows discriminate between mining and plant costs and revenues. It is clear that what is going to give profits is the fact that there is a metallurgical process, which is the path to get a final product of so much worth than the raw material gotten from the very mine. Then it is needed to have the mining costs and the metallurgic costs, also, the revenues of the selling of the gold to the market after the metallurgic process has been completed, which is the net smelting return, NSR [34]. The cash flows can be found by using Eq. (16).

$$F_k = NSR_k - VC_{m,k} - FC_{m,k} - OC_{m,k} \tag{16}$$

where $F_k$ is the net cash flow in year $k$; $NSR_k$ the Net Smelter Return in year $k$; $VC_{m,k}$ the variable operating cost of mining in year $k$; $FC_{m,k}$ the fix operating cost of mining in year $k$; $OC_{m,k}$ the overhead operating cost of mining in year $k$; $m$ subindex for the mining associated costs in year $k$. The variable mining costs can be calculated by means of Eq. (17):

$$VC_{mk} = VOC_k * T_0 + DC_k + BC_k + LC_k + HC_k + CC_k + FCC_k + GSC_k + PC_k + TC_k + AC_k + CEP_k \tag{17}$$

where $VOC_k$ is the variable cost per tonne of ore extracted; $T_0$ the production rate of ore; $DC_k$ the drilling cost; $DC_k$ the blasting cost; $LC_k$ the loading cost; $HC_k$ the haulage cost; $CC_k$ the crushing cost; $FCC_k$ the fine crushing cost; $GSC_k$ the grinding section cost; $TC_k$ the tailing cost; $AC_k$ the assaying cost; and $CEP_k$ the cost of electric power. The fixed cost in year $k$ can be found with Eq. (18).

$$FC_{m,k} = FOC_k + GSC_k + SC_k + ATC_k \tag{18}$$

where $FOC_k$ is the fixed operating cost in year $k$; $GSC_k$ the general service cost in year $k$; $SC_k$ supervision, maintenance and general cost in year $k$; and $ATC_k$ the administrative total cost in year $k$. The overhead operating costs of mining at year $k$ can be found by means of Eq. (19).

$$OC_{m,k} = EC_k + GSC_k + PSC_k + AC_k \tag{19}$$

where $EC_k$ is the engineering overhead cost in year $k$; $GSC_k$ the general site overhead cost in year $k$; $PSC_k$ the project supervision overhead cost in year $k$; and $AC_k$ administrative overhead cost in year $k$.

The IRR is calculated by the use of numerical methods of try and error, for it has to be cleared from Eq. (20):

$$\sum_{k=1}^{N} \frac{F_k}{(1+IRR)^k} = I_0 \tag{20}$$

### 2.6. Genetic algorithms

A genetic algorithm (GA) is a metaheuristic inspired in evolution. In the case of this paper, GAs are to be seen as optimizers, for the goal in a mining project from the money point of view, is to optimize the NPV, it means, to maximize it.

In a broader use of the term, GA is any population based model that uses selection and recombination operators to generate new sample points in a search space. So most of the GA models are introduced from an experimental perspective, it is, from mathematical models that connect parameters and objective functions, so the GA is a tool to optimize those objective functions [16].

Genetic algorithms are also used to resolve very complex and computational demanding problems of optimization, by making them more efficient and accurate, and it involves among other techniques, the one known as parallel computing [35].

#### 2.6.1. Steps of a GA

The steps a GA follows and the way they were implemented in this case of study, are basically the next ones: generating an initial population, calculating the function evaluation, calculating each parameter value probability of survival, drawing with replacement, a new population based on the probabilities calculated on step 3, performing crossover, performing mutation, reinserting best known solution into population at given intervals, and repeating step 2 until stopping criteria is met [35].

## 3. Case of study: hypothetic mine

### 3.1. How is a GA to be applied

In the case of study of this paper, every one of the steps a GA is applied as follows:

(1) Generate an initial population: data is collected from acknowledged sources about the parameters that matter most in the NPV of a mining project. These parameters are chosen because of documented experience.
(2) Calculate the function evaluation: NPV is the variable to optimize, and to calculate it, there are several tools used in this study, like the O'Hara costing model, net smelter return for metallic projects, financial techniques and Montecarlo simulation, and all they will be explained further in this paper.
(3) Calculate each parameter value probability of survival: probability distributions are assigned to each of the inputs, based on authors data about the kind of mining project that is intended to be studies in this paper.
(4) Draw with replacement, a new population based on the probabilities calculated on step 3. Montecarlo simulation is the methodology used in this case to achieve this step. It supports on the deterministic models used in this work, i.e., O'Hara costing model, NSR, and the projection of cash flows to calculate the NPV.
(5) Perform crossover: by means of the deterministic models considered, both kinds of variables, inputs and outputs are interconnected, and by using a Montecarlo simulation considering the variability of all inputs, plus the data acquired from authors who researched about real mining projects like the hypothetical one analyzed here, the crossover of multiple data is performed.
(6) Perform mutation: for the purposes of this work, mutations are performed by considering two case scenarios: low certainty, which implies uniform probability distributions, and a high certainty scenario where normal and other kinds of probability distributions are considered, for the inputs to affect the NPV.
(7) Reinsert best known solution into population at given intervals: for every iteration performed in Montecarlo simulation, the NPV's sensitivity in relation to each of the inputs is recalculated and refined.
(8) Repeat step 2 until stopping criteria met: this is done for both of the case scenarios mentioned, and iterations take place within Montecarlo simulation. Every of the simulations performed have 10,000 iterations. Repeating step 2

means that new values are generated for the inputs, to execute then simulation of the NPV again. The stopping criterion met when all iterations are performed.

So, for this work, 2 mutations or case scenarios were studied, 10,000 iterations each, in order to analyze different scenarios and compare them, one which represents a low certainty scenario and other a high certainty scenario. This is to be explained further.

This large amount of iterations assures that almost every of the simulations for the same case scenario would throw true and trustworthy results for the objective functions, for one simulation itself, is of great confidence under these conditions.

### 3.2. Parameters and data

A gold mine project has a level of risk in relation to its NPV, depending on the level of knowledge that investors may acquire about it. Operational and economic matters about the mine project have to be investigated as thoroughly as possible, in order to assess the real risk of investing money in it, to take it through all the phases it requires to be carried out successfully and yield net profits.

Technical information like ore grade, overburden, inter-burden, stripping ratio, recovery rate; and financial and mineral market information like ore concentrate prices, free risk rate, debt costs, etc., has to be gathered to measure how risky a gold mining project is (Table 1).

The input data for simulating the NPV, IRR and WACC is in hand. Having the mean value of every input variable, the corresponding standard deviation is estimated by taking the ten percent of the former (except for the gold price, whose standard deviation was found from history data), as suggested by the Palisade Risk Industrial software by default, it is shown in Eq. (21):

$$\sigma = 0.1\mu \tag{21}$$

As mentioned before, two scenarios are to be studied, one in which uncertainty is high because of the lack of knowledge of both, the gold deposit and mineral markets, and other in which there is more certainty about them.

The real point is to see how these two circumstances affect the output variables named before, it means, to assess the risk of investing in the project under every of the named circumstances. Input data for simulating a gold mine project's outputs NPV, IRR and WACC under a lower and higher level of certainty scenarios is available in Table 1, it means that very little of reliable information is available and both the market and mine have been studied less deeply.

### 3.3. Low certainty scenario

A low level of certainty circumstance is produced because of the lack of information about the project, and produces a vision of the project to be of high risk from the financial point of view, because there is a probability of 43.9% of getting a negative NPV, as the simulation shows (Fig. 2). A 90% confidence interval between −$0.88 and 3.72 billion dollars is set under low certainty conditions.

The value of assessing the mine and minerals market is clear here, in order to increase the probability of success or having certainty about looking for another investment option different than the open pit gold mine. This high lack of knowledge of the mine and market is proposed in this paper to be modeled by using uniform distributions of probability.

The reason why it is modeled like this is because it fits better with the reasonable considerations that are taken when thinking of scenarios for the project, there is no clear knowledge of which possible scenarios are more or less probable to happen, so all of them get assigned the same probability.

For instance, it is known that the real gold price (adjusted to 2015) historically from year 1980 to 2012, i.e., from pike to pike, has had a minimum of $355 USD/oz. and a maximum of 1897 USD/oz, and because of the lack of more information, the Probability Distribution Function (PDF) assigned to this variable is uniform (Fig. 3).

A low certainty scenario means that a few of reliable information is available and both, the market and mine have barely been studied. In other words, this is a prefeasibility stage of the project assessment. The probability distributions of NPV, IRR and WACC under these conditions can be seen in Figs. 2, 4 and 5.

In Fig. 6, there are box plots for both, IRR and WACC, in order to see if there are chances for them to overlap. In Fig. 7, it is reported the sensitivity of the NPV to the inputs considered in this study, it can be observed that the price of gold is the variable to which the NPV is most sensitive, followed by the average ore grade of the mine.

This prefeasibility stage, motivates or not to continue doing the rest of the job, which is, finding more reliable information to assess the NPV again.

At this stage of the Project evaluation and assessment, information has to be gathered:

(1) Exploring the deposit is an important task to perform, so as to have clear the reserves of the ore within the terrain to be mined, and to assess the concentration of the ore of economic interest within the deposit.

**Table 1**
Input variables for the O'Hara cost model.

| Input variable | Mean | Standard deviation | Low certainty scenario distribution | High certainty scenario distribution |
|---|---|---|---|---|
| Stripping ratio (SR) [33] | 13.57 | 1.357 | Uniform | Normal |
| Tons of ore mined/day $T_0$ (tons/day), [34] | 3000 | 300 | Uniform | Normal |
| Price*P* (USD/oz) [37] | 794 | 402 (from history data) | Uniform | Truncated risk log logistic |
| Average grade of ore in rock (g/ton) [33] | 21.00 | 2.1 | Uniform | Normal |
| Recovery rate of the metallurgical process ($R_{r,k}$) [33] | 95% | 9.5% | Truncated uniform | Truncated normal |
| Stripping soil overburden, $D_0$ (m) | 21.33 | 2.133 | Uniform | Normal |
| Tax rate *T* (yearly) | 4.1% | 0.41% | Uniform | Normal |
| Debt cost *kd* (yearly) | 4.17% | 0.417% | Uniform | Normal |
| Proportion of the financing taken provided by preferred stock, *Wpfd* | 5.52% | 0.552% | Uniform | Normal |
| Debt/equity ratio | 14.10% | 1.41% | Uniform | Normal |
| Dividend/share price (market risk rate), *Kpfd* | 5.52% | 0.52% | Uniform | Normal |
| Risk free rate (yearly), *Rfr* | 0.12% | 0.012% | Uniform | Normal |
| Beta for the mining industry | 128.0% | 12.8% | Uniform | Normal |

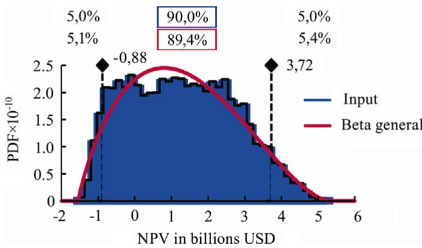


**Fig. 2.** Probability Distribution Function (PDF) of the NPV under a low certainty scenario.

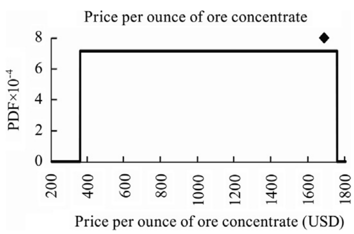


**Fig. 3.** Probability distribution for real gold prices in USD/Oz under a low certainty scenario.

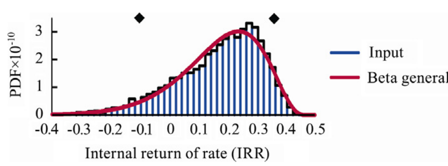


**Fig. 4.** Probability Distribution Function (PDF) of the IRR under a low certainty scenario.

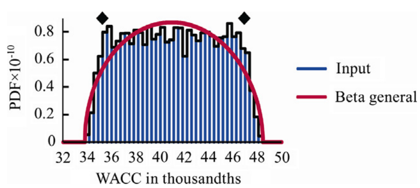


**Fig. 5.** Probability Distribution Function (PDF) of the WACC under a low certainty scenario.

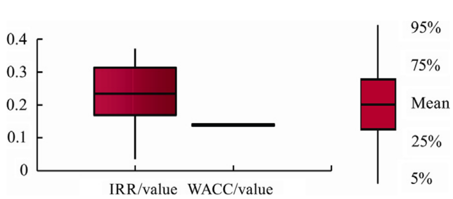


**Fig. 6.** Box plot of IRR and WACC under a low certainty scenario.

(2) Mineral markets have to be investigated (gold markets and its possible substitutes).
(3) Technical matters have to be specified for the deposit that is being investigated for both mining and metallurgical processes (ore grade distribution in the deposit, recovery rate, etc.).
(4) And also, a thorough investigation on financial markets has to be done, so as to establish optimal debt-equity ratios to be taken, debt costs, risk free rates, etc.

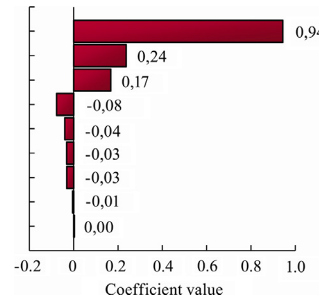


**Fig. 7.** Sensitivity of the NPV to the inputs under low certainty conditions.

All the items named before, imply, for the interested parties, the investment of capital.

### *3.4. High certainty scenario*

A higher level of certainty in the project data reduces the possible scenarios to be considered, and even better, makes more clear the real chances investors are taken about the gold mine they are putting their money in. There is no wonder that things get clearer than in the previous case, even though there are still uncertainties to be taken into account. The financial variables NPV, IRR and WACC of the project are expected to have less variance, and it is very important for decisions taken under a more real set of scenarios, discarding the ones that definitely are less probable to happen.

#### *3.4.1. Market study: gold cycle*

One thing that in a gold mining project is to be studied with depth, is the gold market itself, as it has been proved in the low certainty scenario, for its strong impact on the NPV. Gold is a commodity, as it is known, so, it has a cyclic behavior [18].

Because of the world economic crisis, banks around the world were demanding high amounts of gold until 2012 when the gold price bubble seemed to burst. For they in the 90s sold most of their gold reserves, because of its low price, and in consequence the gold pattern for currencies was removed, so the stock market was more attractive at that time.

Nowadays, on the contrary, stock market leaves most of the investors with serious doubts, the European and USA high external debt increase those doubts, producing in consequence, a higher demand for gold, which has been the reason why gold prices are having an uptrend from year 2000 to 2012 [19]. Today gold prices have been lowered with respect to 2012 and are apparently stagnant for the slight recovery the USA economy has have, but doubts still remain. The Gold price study tries to establish a range of possible prices for gold. Nominal prices were gathered, and then they were adjusted by using Eq. (22) [37].

$$p_{r,y_b} = p_{n,y_0} \prod_{r=y_0}^{y_b} (1 + i_r) \tag{22}$$

where $P_{r,yb}$ is the real price adjusted to year $y_b$; $P_{n,y0}$ the nominal price on year $y_0$; $y_b$ the base year (2015 for this study); $y_0$ the year in which the nominal price took place; $i_r$ the inflation rate at year $r$; $r$ a counter that starts in year $y_0$ and ends in year $y_b$.

In Fig. 8, the gold cycle has been characterized. The data taken to build a probability distribution are the real prices of gold, adjusted to year 2017 by means of the USA inflation. Though its high prices, record price of 1980 was not reached yet in year 2012, where there was another pike for the real gold prices in the last 35 years [37]. It makes sense according to, who calculated that the Dow-gold ratio cycle takes an average of 37 years, i.e., from one pike to the next [21].

To simulate a more deep knowledge scenario, normal distributions have been assigned to all of the input variables, leaving parameters to remain the same as in the low certainty scenario, but gold prices. Truncation on gold prices was made to ensure the software would iterate with prices from up to $355 USD/oz. for it was the minimum price that took place within the years 1980 and 2012, i.e., from pike to pike (Fig. 9). It really gives a more deep knowledge of the market to the analysts.

A probability distribution was adjusted to real gold prices by using the Palisade Risk 5.5 industrial distributions portfolio. It yielded a risk inverse Gauss distribution to be the best fit one, and data from 1980 to 2012 was used. Taking a look at Fig. 8 for details about this. Under this level of knowledge of the project (mine and market), 10,000 iterations took place when simulating in Palisade Risk Industrial software.

### 3.5. Geological exploration and data

Exploration is needed to be carried out within the subsoil of the future mine, in order to get more information about some technical matters that will have effect on the exploitation phase when it begins to take place, mainly the ore grade, but also the rock hardness, interburden distance, overburden distance, topography, etc.

To augment the certainty of the knowledge of the ore grade, a more close grid has to be designed, followed by the boring of holes, the logging of the field data (topographical coordinates) and further geochemical analysis performed by certified laboratories, which complies with the accepted international reporting standards: NI-43/101, JORC; SAMREC [25]. Geostatistics have to be used for analyzing the data log gotten from the laboratory analysis, techniques like Krigging and recently neural networks combined with fuzzy logic are used for estimating the ore grade of the mine based on the logged data, in which it is contained both, topographical coordinates and ore grade [23,39]. This detailed analysis allows also the achieving of a probability distribution of the ore grade varying it throughout the life of mine.

There was not found a specific probability distribution that characterizes the ore grade of a gold mine through time by experience in the literature by now. But every author agrees in the fact that when the mine starts its exploitation phase, the higher grade rocks have to be extracted and processed first, in consequence, the ore grade of the mine will fall as the extraction goes on, until it will be less than the cutoff grade, which will lead to the closure of the mine.

Then, a normal distribution is assigned to this variable for simulating a high certainty scenario, and supported on surface mining projects, the mean ore grade for surface gold mines is taken as 21 g/ton [33].

### 3.6. Output: NPV, IRR and WACC

The probability distributions for NPV, IRR and WACC can be seen in Figs. 10–12. Now a 90% confidence interval between −$1.08 and 2.32 billion dollars is reported for the NPV, where there is a 59.2% probability of getting a negative value. In Fig. 13, there are box plots for both, IRR and WACC, in order to compare them and see if there are chances for them to overlap. In Fig. 14, it is reported the sensitivity of the NPV to the inputs considered in this study, and it can be observed that the price of gold is the variable to which the NPV is most sensitive, followed by the average ore grade of the mine.

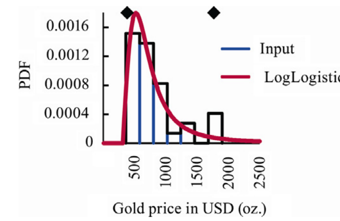


**Fig. 9.** Inverse Gauss distribution adjusted to real gold prices in 2017 US dollars.

## 4. Discussion of results

### 4.1. Sensitivity analysis

Interpretation of the sensitivity analysis is required, in order to establish how to maximize the NPV around an operating point like the one shown in Table 1.

Sensitivity analysis was performed under two case scenarios for the NPV, as shown in Figs. 14 and 7. They both show a high sensitivity to the price of gold, and it is expectable to be that way, for this variable affects the operational income directly and immediately, and because it is very volatile and is out of the control of both investors and mine operators, and it introduces the most risk to the NPV. This fact leads to a volatile NPV too, which is not ensured to be positive under the conditions simulated, and strategies for maximizing its value have to be implemented.

That analysis leads to the linear relations shown in Eqs. (23) and (24), taken from Figs. 7 and 14.

$$NPV_h(bill.USD) = 0.94P + 0.22g + 0.17R_r - 0.11T_0 - 0.05SR - 0.04D_0 - 0.01K_{pfd} \quad (23)$$

$$NPV_l(bill.USD) = 0.94P + 0.24g + 0.17R_r - 0.08T_0 - 0.04k_d - 0.03SR - 0.03D_0 - 0.01K_{pfd} \quad (24)$$

where $NPV_h$ is the net present value under high certainty conditions in billions of U.S. dollars; $NPV_l$ the net present value under low certainty conditions in billions of U.S. dollars; $P$ the price of gold in USD/oz; $g$ the mean ore grade of the mine throughout its life in g/ton; $R_r$ the recovery rate of the metallurgical process; $T_0$ the tones of ore mined per day; $k_d$ the debt cost; $SR$ the stripping ratio of the mine; $D_0$ the stripping soil overburden in feet; and $K_{pfd}$ the dividend/share price ratio or market risk rate.

This relationships are valid for the inputs moving within their given intervals, as they were set in Table 1.

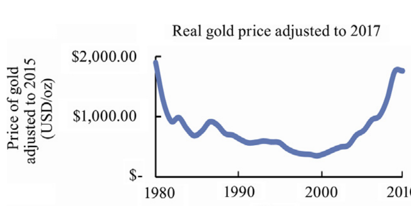


**Fig. 8.** Gold price study to establish trends of its real price, adjusted by inflation in the USA to year 2017 [37].

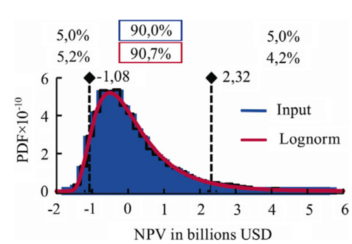


**Fig. 10.** Distribution of the NPV under a high certainty scenario.

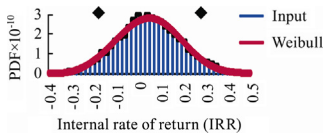


**Fig. 11.** Distribution of the IRR under a high certainty scenario.

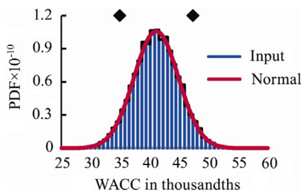


**Fig. 12.** Distribution of the WACC under a high certainty scenario.

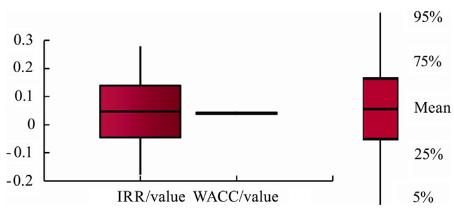


**Fig. 13.** Box plot of IRR and WACC under a high certainty scenario.

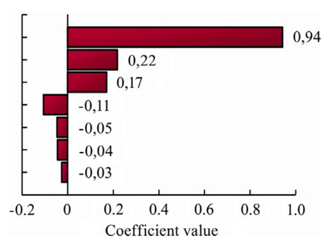


**Fig. 14.** Sensitivity of the NPV to the inputs under high certainty conditions.

On the contrary of what many may think, reducing the rate of rock mined daily to the minimum value of its confidence interval, is a way to increase the NPV, and it is understandable in the sense that costs will be reduced, and profits will be augmented by focusing in the processing of higher grade rock. It could be achieved by augmenting in the right proportion the cutoff grade of the mining project, and further study have to be made in order to optimize it. Different techniques are used to optimize the cutoff grade, like utility functions, among others [22,24,26,39].

## 5. Conclusions

The first thing to be analyzed in order to see if a gold mining project could be profitable is its equilibrium point. And the demand has to be far superior to this point, in order to be able to cover with the sells both, fixed and variable costs, and with time, to cover the initial investment, to finally generate a positive NVP.

The order of importance of the impact on the NPV of the input variables studied is the same in both cases, low certainty and high certainty of the information available. This fact makes a strong statement that contributes to the experience gathered about open pit gold mining projects in that sense. And future research is to be carried out for gold small scale mining projects, where investment capital is more limited than in the kind of large mining projects studied here, and decisions are made under low certainty circumstances, regarding the information available. These findings lead to propose for small scale miners to focus in market research strategies and a thorough geological exploration of their deposits, so as to have higher certainty information and therefore, to assess better these kind of projects in terms of their economic feasibility.

The real price of gold is the most influential parameter on the mining project's NPV, and the fact that this one is out of control for both, investors and mining operators, to go ahead with a project of this kind of project turns always to be a risky endeavor. There is no wonder that today, gold prices are not as high as they were in 2012, also it is known that the world economic recession of 2008, which has not been fully overcome yet, made commodities like gold to show an uptrend since year 2000. But as economic bonuses are not forever, recessions are not either, so gold prices are cyclic as the economy is, and in the next 5 years they would change the way they behave dramatically, keeping the uptrend or falling to levels seen in the 90s. Because of the high uncertainty about gold prices, many new gold mining projects around the world have been delayed or suspended, until that situation improves well enough to go ahead.

Because the ore grade is the second most influential variable on the NPV, exploring the mine have becomes a very thorough work to be done, according to strict standards, and checking the logged data and resource valuations by QPs, which ensure the reported data is trust worthy.

Increasing the recovery rate is also an increase for the NPV, so the metallurgical process has to be watched carefully during all the life of mine, also ensuring a good design of the plant from the very beginning, so as to make the benefit process as highly efficient as possible.

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